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Predictability of atmospheric boundary-layer flows as a function of scale

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[1] Predictability analysis based on the maximum Lyapunov exponent considers infinitesimal perturbations, which are associated with errors in the smallest fastest-evolving scales of motion. However, these errors become irrelevant for the predictability of larger scale motions. In this study we employ the newly developed Finite Size Lyapunov Exponent (FSLE) analysis to assess predictability of atmospheric boundary layer flows as a function of scale. We demonstrate the expected enhanced predictability at large scales and quantify the dependence of predictability on the stability of the atmospheric environment. *INDEX TERMS:* 3379 Meteorology and Atmospheric Dynamics: Turbulence; 3220 Mathematical Geophysics: Nonlinear dynamics; 3307 Meteorology and Atmospheric Dynamics: Boundary layer processes; 3240 Mathematical Geophysics: Chaos. **Citation:** Basu, S., E. Foufoula-Georgiou, and F. Porté-Agel, Predictability of atmospheric boundary-layer flows as a function of scale, *Geophys. Res. Lett.*, *29*(0), XXXX, doi:10.1029/2002GL015497, 2002.

1. Introduction

[2] Atmospheric boundary layer turbulence possesses many scales of motion not all of which are resolved in numerical prediction models of the atmosphere. When numerical prediction models are used to assess the predictability of atmospheric flows (via identical twin experiments), the perturbations of the initial conditions are of finite size and apply to scales of motion that are larger than the smallest characteristic scales of the atmosphere. In this case, the standard maximum Lyapunov exponent analysis has been found to give predictability times much smaller than those inferred by verification studies [*Lorenz*, 1996]. This is because the effect of the smallest fast-evolving scales (considered by the maximum Lyapunov exponent analysis) becomes irrelevant for the predictability of larger-scale motions. To address this problem *Aurell et al.* [1996a] introduced the concept of Finite Size Lyapunov Exponent (FSLE) that becomes particularly useful when there exists a hierarchy of characteristic scales such as the eddy turnover times in three-dimensional fully developed turbulence. FSLE has been verified in the literature through various numerical experiments (coupled map lattices, shell models and toy models of atmosphere) [see, e.g., *Aurell et al.*, 1997; *Boffetta et al.*, 1998a, 1998b] but, to the best of our knowledge, has not been exploited yet on actual observations or numerical weather prediction model outputs to assess atmospheric predictability as a function of scale.

[3] In this paper, we use long records of high-resolution atmospheric turbulence data to quantify the predictability of atmospheric boundary layer flows over a homogeneous terrain. The results clearly demonstrate that predictability strongly depends on the scale at which the process is considered with larger predictability at larger scales. They also demonstrate that the degree of predictability varies according to the stability of the atmospheric conditions. In general, the more stable the environment the more predictable it is (at least over a wide range of prediction error tolerances), a result that would be difficult to obtain from the maximum Lyapunov exponent analysis.

2. Finite Size Lyapunov Exponent

[4] In chaotic dynamical systems sensitivity to initial conditions limits the predictability time to:

$$T_p \sim \frac{1}{\lambda_{\max}} \ln\left(\frac{\Delta}{\delta}\right) \tag{1}$$

where $\lambda_{\max}$ denotes the maximum Lyapunov exponent, δ is the size of initial perturbations and Δ is the prediction error tolerance. The above formula holds only for infinitesimal perturbations and in nonintermittent systems. *Aurell et al.* [1996a] introduced a generalization of the maximum Lyapunov exponent to allow for finite perturbations relevant to the predictability of complex systems comprising of a wide spectrum of scales. This new measure, properly named as "finite size Lyapunov exponent" (FSLE) is based on the idea of error growing time $T_r(\delta)$, which is the time it takes for a perturbation of initial size δ to grow by a factor r (equals to $\sqrt{2}$ in this work). The FSLE is then defined as

$$\lambda(\delta) = \frac{1}{\langle n_r \rangle} \left\langle \ln\left(\frac{\delta(n_r)}{\delta}\right) \right\rangle \tag{2}$$

where $\delta(n_r)$ is the size of the perturbation at the time n_r at which this perturbation first exceeds (or becomes equal to) the size $r\delta$. The average $\langle \ldots \rangle$ is over an ensemble of many realizations. In this case, for an initial error δ and a given tolerance Δ, the average predictability time can be written as:

$$T_p = \int_{\delta}^{\Delta} \frac{d \ln \delta'}{\lambda(\delta')} \tag{3}$$

In the limit of infinitesimal perturbations ($\delta \rightarrow 0$), $\lambda(\delta)$ approaches $\lambda_{\max}$ and T_p takes the usual form (equation 1) [see *Aurell et al.*, 1996a for details].

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[5] It is seen that for systems which possess many scales of motion, from the smallest fast evolving low-energy containing scales to the largest slow-evolving high-energy containing scales, λ_{max} does not characterize the predictability of the system. It only characterizes the growth of infinitesimal errors associated with the smallest scales. Larger errors, which typically occur at larger scales (for example, at the grid scale of a numerical model) will grow with a different rate as prescribed by the FSLE $\lambda(\delta)$.

3. Analysis of Atmospheric Turbulence Data

[6] Data sets from a field experiment carried out over an unobstructed flat terrain at the Campbell Tract research field of the University of California at Davis during the summer of 1999 were used in this study. The basic instrumentation setup consisted of twelve three-dimensional sonic anemometers arranged in two parallel horizontal arrays to simultaneously measure the longitudinal, lateral and vertical wind velocity components as well as the air temperature at a sampling frequency of 20 Hz. The details of the setup can be found elsewhere [*Porté-Agel et al.*, 2001]. Four longitudinal velocity time series (will be referred to as u_A, u_B, u_C and u_D) corresponding to four different atmospheric stabilities (stable, moderately stable, weakly stable, unstable: Obukhov length of 1.36 m, 9.16 m, 196.36 m and −31.71 m, respectively; see *Stull*, 1988 for the definition of Obukhov length) were analyzed for predictability (Figure 1). All the series were collected from one particular anemometer (sensor height = 3.41 m) and their length was approximately 35,000 corresponding to 30 minutes of experiment period.

[7] Since nonlinearity is a necessary condition for the presence of deterministic chaotic dynamics [*Diks et al.*, 1995], detection of nonlinearity is an obvious prerequisite to avoid spurious computation of typical invariant measures (e.g. dimensions, Lyapunov exponents, entropies etc.). In this work we tested for nonlinearity using a newly proposed framework [*Basu and Foufoula-Georgiou*, 2002], which is based on probabilistic comparison of the phase-space of the original series and a number of linear stochastic surrogates. The metric used for comparison of the phase spaces is the "transportation distance" function [*Moeckel and Murray*, 1997] which accounts for both geometric and probabilistic factors (thus superior to the "total variation distance"), is less sensitive to outliers, noise and discretization errors (thus better than the "Hausdorff distance") and measures the long-term qualitative differences (differences in the dynamics) between two series (thus preferable to simple correlation coefficient or root-mean-squared error). The set of transportation distances is first computed between the original series $\{x_n^o\}$ and all the surrogates $\{x_n^i\}$, $i = 1, \ldots, N_s$:

$$D_{OS} = \left\{ d_{oi}\left(x_n^o, x_n^i\right),\ i = 1, \ldots, N_s \right\} \tag{4}$$

In a similar way, the set of mutual distances between the surrogates are computed:

$$D_{SS} = \left\{ d_{ij}\left(x_n^i, x_n^j\right),\ i,j = 1, \ldots, N_s \text{ and } i \neq j \right\} \tag{5}$$

If the frequency histograms of D_{OS} and D_{SS} are roughly nonoverlapping, the null hypothesis of linear stochasticity can be rejected, and the original data can be considered to be nonlinear at this significance level. A simple measure of the "Degree of Nonlinearity"(DON) can be written as:

$$DON = \frac{mean(D_{OS}) - mean(D_{SS})}{mean(D_{SS})} \tag{6}$$

In this paper, all the series were embedded in phase space of dimension three and compared with nine surrogates with the help of the transportation distance function (please refer to *Basu and Foufoula-Georgiou* [2002] for technical details). All series were found to exhibit nonlinearity (u_A, u_B, u_C and u_D: DON of 53, 60, 89 and 68 percents respectively). Generally, the degree of nonlinearity was found to increase with the degree of instability with the exception of the weakly stable series u_C indicating stronger nonlinearity than the unstable series u_D.

[8] After detecting nonlinearity, the series were analyzed by the FSLE algorithm to infer predictability. For the application of FSLE algorithm the choice of optimal embedding is crucial. Given an original scalar series, a suitable delay time and an embedding dimension were chosen following the ideas of mutual information and false nearest neighbors [*Kantz and Schreiber*, 1997] respectively. For the sake of brevity a detailed description of the computation of FSLE from measured data is not covered here. Interested readers are referred to the paper by *Boffetta et al.* [1998b]. The implementation of the algorithms (mutual information, false nearest neighbors and FSLE) were done using the TISEAN package [*Hegger et al.*, 1999]. Figure 2 shows the plot of $\lambda(\delta)$ as a function of perturbations δ for all the series. It is seen, as was expected, that for smaller δ, $\lambda(\delta)$ reaches a plateau, implying that the magnitude of λ_{max} is approximately 3. Interestingly, all the δ vs. $\lambda(\delta)$ curves can be closely approximated by the following simple relation:

$$\lambda(\delta) = \frac{1}{a + b\delta^2} \tag{7}$$

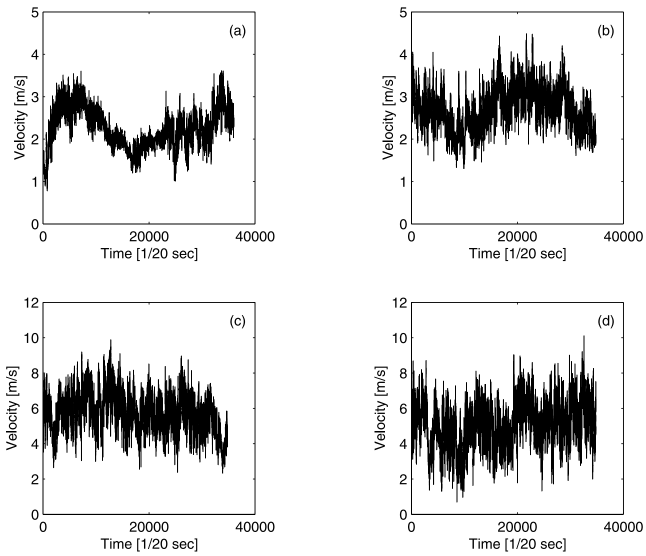

Figure 1. Time series of the longitudinal velocity component for (a) stable, (b) moderately stable, (c) weakly stable, and (d) unstable atmospheric conditions.

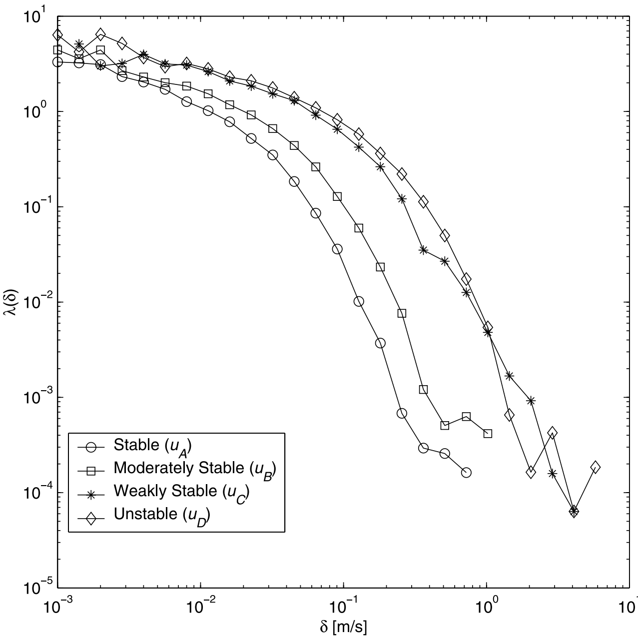

Figure 2. Finite size Lyapunov exponent, $\lambda(\delta)$, as a function of the size of initial perturbations, δ, for all longitudinal velocity time series.

which can be seen as providing a concise parameterization of $\lambda(\delta)$ as a function of δ (see, e.g. Figure 3 for the fit of equation 4 to series u_A).

[9] To verify our findings, we used a standard algorithm [*Rosenstein et al.*, 1993] to compute $\lambda_{\max}$ directly from the observed series. Figure 4 shows the typical plot of the number of iterations versus the natural logarithm of error divergence (the slope is the maximum Lyapunov exponent if it is a straight line). Evidently, the straight line segments correspond to exponential growth driven by different dominant scales and the global (envelope) evolution is not

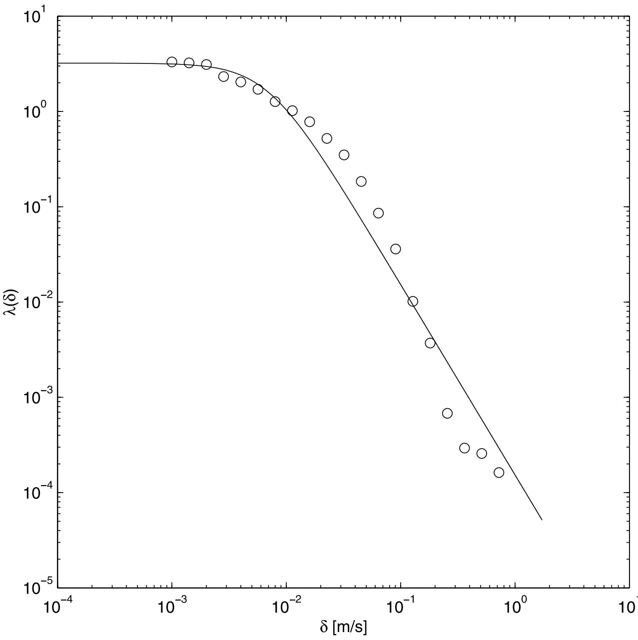

Figure 3. $\lambda(\delta)$ vs. δ for series u_A and the fitted line corresponding to equation (7).

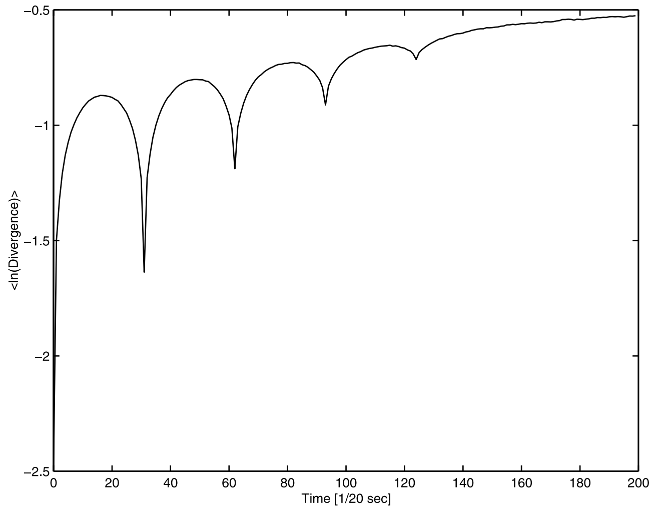

Figure 4. Typical plot of $< ln(\text{divergence}) >$ versus time for the time series u_A (the slope is the maximum Lyapunov exponent if it is a straight line). Note that the slope exhibits no global linear increase behavior.

exponential but closely follows a power law as argued by *Aurell et al.* [1996b]. Figure 1 of *Aurell et al.* [1996b] is a similar plot for a chain of coupled maps in the context of predictability in systems with many characteristic time scales. The above issues question earlier predictability studies based on standard Lyapunov exponent for turbulent flows and force us to acknowledge the need of FSLE for turbulence data including the estimation of $\lambda_{\max}$.

[10] Figure 5 shows the predictability of u_A (mean of 2.26 m/s and standard deviation of 0.46 m/s) as a function of

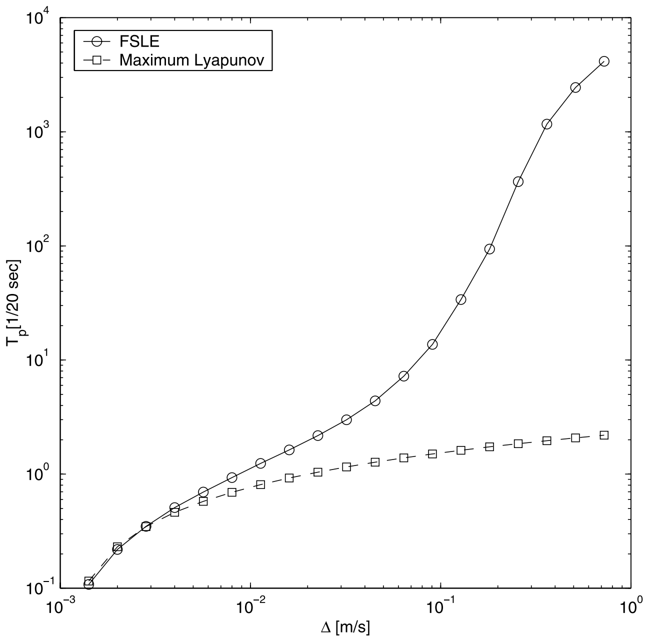

Figure. 5. Predictability time T_p based on FSLE and Maximum Lyapunov Exponent for the series u_A (initial error of 10^{-3} m/s). Note that predictability inferred by assuming infinitesimal perturbations (dashed line/squares) is much smaller than that inferred by finite-size perturbations (solid line/circles).

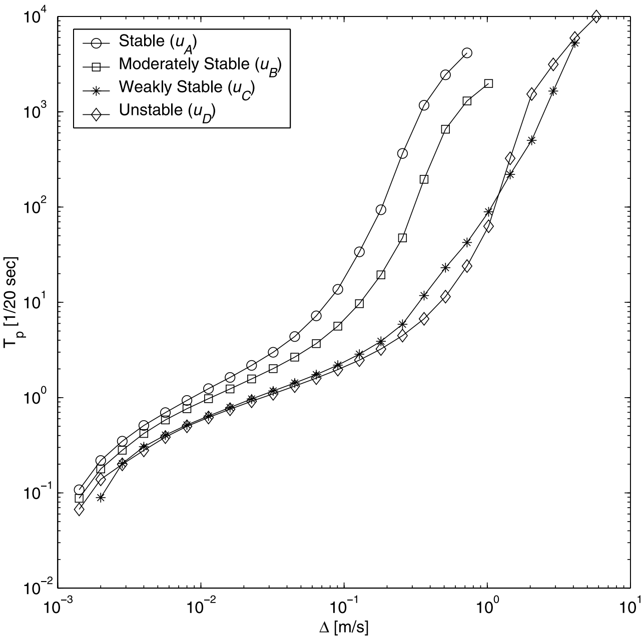

Figure 6. Effect of atmospheric stability on predictability for a range of prediction error tolerances Δ. Note that the range of error tolerances is indirectly related to the standard deviation of each series and thus is higher for u_C and u_D. The initial error δ was 10^{-3} m/s.

tolerance Δ. Observe that, for an initial error of 10^{-3} m/s, the maximum Lyapunov exponent analysis shows that this error will grow to 10^{-1} m/s after approximately 0.1 seconds ($2 \times 10^0 \times 1/20$), at which time the predictability of the system is almost lost (see the dashed line/squares). On the other hand, the FSLE analysis (which is more appropriate for an initial perturbation of 10^{-3} m/s, and error growth of the order of 10^{-2} to 10^{-1} m/s, which are finite with respect to the aforementioned statistical properties of the series) shows that predictability is indeed much larger. For example, it shows that an initial error of 10^{-3} m/s will grow to 10^{-1} m/s, in 1 second (10 times larger than what the λ_{max} analysis shows) and will grow to 1 m/s (an error of the order of the velocity itself, but still of practical interest for large-scale motions), in 100 seconds. The interplay of δ and Δ on determining the predictability time is nicely seen in the following equation easily derivable from equation (3) using equation (7):

$$T_p = \frac{1}{\lambda_{max}} \ln\left(\frac{\Delta}{\delta}\right) + \frac{b}{2}\left(\Delta^2 - \delta^2\right) \tag{8}$$

In agreement with the previous observations, one can see from (5) that when Δ is in the order of δ, the first term of the above equation is dominant; on the other hand when Δ is much larger than the initial perturbation, the second term becomes much more prevailing (λ_{max} is roughly proportional to the inverse of the smallest low-energy-containing time scales, and, therefore, does not play any role in the predictability of large scale motions).

[11] Furthermore, it is interesting to note (from Figure 6 and also from Figure 2) that predictability depends on the stability of the atmospheric turbulence environment. Intuitively, the more stable the environment, the more predictable it is expected to be. This is seen from Figures 2 and 6 with the exception of the weakly stable series u_C which was found less predictable than the unstable series u_D for high prediction error tolerances ($\Delta > 1$ m/s). This anomaly found from only one series calls for further investigation. It is noted that the dependence of predictability on the stability of the atmospheric environment would be difficult to infer with the standard maximum Lyapunov exponent analysis since all four curves would saturate to almost the same predictability time (due to similar values of λ_{max}). Indeed, nonlinear regression of the δ vs. $\lambda(\delta)$ curves for the four series yielded very similar values of λ_{max} (in the range of 3.2 to 4) but diverse values of the parameter b (u_A, u_B, u_C and u_D: b equals to 191, 226, 1912 and 6528 respectively) reflecting different stability regimes. The effect of atmospheric stability on nonlinearity and predictability is an intriguing issue which warrants further study.

4. Conclusions

[12] In this study, we implemented the Finite Size Lyapunov Exponent analysis to assess predictability of atmospheric boundary layer turbulence as a function of scale. Using high frequency horizontal wind velocity data collected at a fixed height over a homogeneous terrain, we demonstrated the enhanced predictability of larger slow-varying scales of motion and the dependence of predictability on the stability of the turbulent environment. Future studies will address the characterization of predictability as related to the multifractal nature of the wind velocity series and also the predictability over heterogeneous terrains as a function of the characteristic scales of heterogeneity and the height of the boundary layer.

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