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Horizontal and vertical self-organization of braided rivers toward a critical state

Victor B. Sapozhnikov and Efi Foufoula-Georgiou
St. Anthony Falls Laboratory, University of Minnesota, Minneapolis

Abstract. Self-organization in an experimental braided river is studied. It is shown that the experimental braided river self-organizes into a critical state where it shows dynamic scaling; that is, small and large parts of the river evolve statistically identically after proper renormalization of space and time. The dynamic scaling emerges during the process of approaching the critical state which involves self-adjustment of both profile (vertical self-organization) and braiding pattern (horizontal self-organization). The obtained result corroborates the hypothesis suggested by the authors earlier [*Sapozhnikov and Foufoula-Georgiou*, 1997] that braided rivers are self-organized critical systems. The results are also important for understanding and statistically predicting the behavior of natural braided rivers because, owing to external conditions (e.g., sudden streamflow changes), some of them may be driven out of the critical state and therefore may show deviation from dynamic scaling.

1. Introduction

Braided rivers are complex systems characterized by hierarchical geometry and rapid evolution. Different approaches have been used to describe and understand their morphology and dynamics. A bulk of research on braided rivers has focused on detailed study of processes in a small area, such as flow and sediment flux around a single channel bar or confluence [*Ashmore and Parker*, 1983; *Ashmore et al.*, 1992; *Best*, 1986, 1988; *Bristow et al.*, 1993; *Mosley*, 1976, 1977; *Robert*, 1993]. Another approach involves exploring statistical properties of the spatial structure and evolution of braided rivers from the point of view of unraveling and quantifying scale relationships in space and time [*Sapozhnikov and Foufoula-Georgiou*, 1996, 1997; *Foufoula-Georgiou and Sapozhnikov*, 1998]. It is hoped that this approach can shed new light on braided rivers as a whole in terms of understanding the underlying mechanisms responsible for the formation and evolution of braided patterns, and statistical prediction of their behavior.

From the analysis of three natural braided rivers, *Sapozhnikov and Foufoula-Georgiou* [1996] showed that these rivers exhibit anisotropic spatial scaling in their static morphology. This implies that a smaller part of a river, stretched differently along the mainstream and the perpendicular directions, is statistically identical to a larger part. Later, *Sapozhnikov and Foufoula-Georgiou* [1997] presented experimental evidence that in addition to the spatial scaling, stationary braided rivers (i.e., rivers given enough time to reach statistical equilibrium under constant external conditions) exhibit dynamic scaling in their evolution. The presence of dynamic scaling implies that a smaller part of a braided river evolves identically (in the statistical sense) to a bigger part provided that time is rescaled by a factor which depends only on the ratio of the spatial scales of the two parts.

The presence of dynamic scaling together with other theoretical considerations led us to conjecture [*Sapozhnikov and Foufoula-Georgiou*, 1997] that braided rivers belong to self-organized critical (SOC) systems [see *Bak et al.*, 1987]. Indeed, in stationary systems, dynamic scaling is a signature of critical state. Also, a large number of degrees of freedom (i.e., a large number of possible independent changes in the patterns) and nonlinearity, both of which are present in braided rivers, are typical of self-organized critical phenomena. SOC systems bring themselves to a critical state which is an attractor of their dynamics. This is different from traditional critical systems, which require external fine tuning of a driving parameter (e.g., temperature in the case of a magnetic material or concentration in the case of a percolation system) to be brought to a critical state [e.g., *Ma*, 1976].

Since by definition SOC systems show critical behavior only after they have brought themselves to a critical state, which is also a statistical equilibrium state, *Sapozhnikov and Foufoula-Georgiou* [1997] left their experimental braided river to evolve until both its profile and braiding pattern reached equilibrium, and then the presence of SOC was tested. We note here that the profile reached the static equilibrium (i.e., it stopped changing), whereas the braiding pattern, while remaining statistically the same, was undergoing continual changes (statistical equilibrium). After the river reached the equilibrium state we analyzed it for criticality and, indeed, found the presence of dynamic scaling, an indicator of a critical state.

However, a more thorough study of a SOC system requires exploration of its behavior not only at equilibrium but also before it reaches this state. Thus, in this study we examine an experimental braided river at different stages, as it approaches statistical equilibrium. There are two motivations for such a study. The first motivation is theoretical. It stems from the fact that critical systems show dynamic scaling at the critical state but deviate from dynamic scaling as they are driven out of this state [e.g., see *Ma*, 1976]. Therefore, to confirm that a state a system brought itself into is critical, it is important not only to demonstrate the presence of dynamic scaling at this state but also to show that the dynamic scaling was not present before and only arose as the system approached this state. The second motivation stems from the fact that in transferring results from

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an idealized experimental environment to natural braided rivers one cannot neglect the fact that a natural river may not be exactly at the equilibrium state. This may be due to natural variability of external factors, as, for example, abrupt discharge changes or long-timescale tectonic events. Another reason is that the river may have changed its path, and therefore, even if the external conditions are stable after the change, the river might simply not have had enough time to reach the equilibrium (this includes slope buildup and reworking external constraints, like mountains). As discussed in section 5, knowing the behavior, in terms of deviation from dynamic scaling, of natural rivers which are not quite at the equilibrium state is needed for statistical prediction of the evolution of such rivers.

In this work an experimental braided river is studied for presence of dynamic scaling at the equilibrium state and before it approached this state. The closeness to the equilibrium state is considered with respect to two factors: profile shape (vertical self-adjustment of the river) and braiding pattern (horizontal self-adjustment). A significantly larger body of data than that used by *Sapozhnikov and Foufoula-Georgiou* [1997] has been obtained and analyzed for the present study.

It should be mentioned that the anisotropy of dynamic scaling, implying that the river evolution scales differently in the direction of the slope and in the perpendicular direction [see *Foufoula-Georgiou and Sapozhnikov*, 1998], has not been considered herein. Rather, we employed the isotropic dynamic scaling framework of *Sapozhnikov and Foufoula-Georgiou* [1997], as it better suits the purpose of this study, which requires establishing presence or absence of dynamic scaling and not evaluating the scaling exponents. Also, the advantage of this framework is that it is more robust and less data-demanding and thus can handle "difficult" cases when the river has approached dynamic scaling but has not quite reached it yet.

2. Review of the Theory

The theoretical framework of testing for the presence of dynamic scaling based on changes in the analyzed objects is described in detail by *Sapozhnikov and Foufoula-Georgiou* [1997]. Here we present only in brief the essence of the theory.

Spatial scaling in a fractal object implies that the object looks statistically the same at different spatial scales. If, additionally, it evolves in such a way that after a proper rescaling of time its evolution is also statistically indistinguishable at different scales, then we say that in addition to spatial scaling, the object exhibits dynamic scaling. The space-time rescaling has the form

$$t_2/t_1 = (L_2/L_1)^z \tag{1}$$

where L_1 and L_2 are the scales at which the evolution of the object is considered, t_2/t_1 is the time rescaling factor, and z is called the dynamic scaling exponent. One can see (1) as providing the space-time rescaling needed to have the projection of the evolution of part $L_1 \times L_1$ on a screen, statistically indistinguishable from the projection of part $L_2 \times L_2$ on a screen of the same size.

Let us characterize the evolution of a stationary fractal object by "changes" in its pattern, where changes are defined as parts of the space which were not occupied by the object at a certain moment of time but became occupied after some time lag t. Let $n(l' > l, t)$ denote the number of changes exceeding size l after some time lag t, and let D be the fractal dimension of the object (e.g., in this work it is the fractal dimension of the braided river spatial pattern). It can be shown [see *Sapozhnikov and Foufoula-Georgiou*, 1997] that the condition for dynamic scaling (1) can be written in terms of the statistics of changes as

$$n(l' > l, t) = l^{-D} f(t/l^z) \tag{2}$$

where $f(\ )$ is some function.

For time lag $t = 0$, there are no changes in the object, which implies $n(l' > l, 0) = 0$, and, correspondingly, $f(0) = 0$. If for small values of the argument the function f can be approximated by a power law, with some exponent β, then the condition (2) for dynamic scaling takes the form

$$n(l' > l, t) \sim t^{\beta} l^{-D-\beta z} \tag{3}$$

Although, in contrast to (2), (3) holds only for small values of t/l^z, it can be conveniently used to facilitate the estimation of the dynamic scaling exponent z. Specifically, the following procedure of estimating z was employed by *Sapozhnikov and Foufoula-Georgiou* [1997]. It was based on their empirical finding that indeed in the experimental braided river, at small t/l^z values,

$$n(l' > l, t) \sim l^{-k} \tag{4}$$

for every fixed value of the time lag t, and

$$n(l' > l, t) l^k \sim t^{\beta} \tag{5}$$

for every fixed value of l.

These two equations coincide with (3) with the dynamic exponent z given as

$$z = (k - D)/\beta \tag{6}$$

Thus, in a system showing dynamic scaling one can first estimate the k exponent from the log-log plots of $n(l' > l, t)$ versus l (for several fixed values of t) and the β exponent from the log-log plot of $n(l' > l, t)l^k$ against t. This, together with the fractal dimension D of the system (estimated using, for example, the mass-in-a-box method; *Mandelbrot* [1982]) enables estimation of z using (6). Then one can plot for the estimated value of z, the values of $n(l' > l, t)l^D$ versus t/l^z, for all (and not only small) values of t/l^z, to verify that the general equation of dynamic scaling (2) holds. Collapsing of all curves to a single curve, the $f(\ )$ curve in (2), would verify the presence of dynamic scaling in the river. The procedure is schematically displayed in Figure 1.

3. Experimental Setup

The experimental setup used to collect the data for this analysis is similar to that described by *Sapozhnikov and Foufoula-Georgiou* [1997], and the reader is referred to that work for more details. Here we present only a brief description and then elaborate on the procedures we used to introduce vertical and horizontal disturbances in the system and to monitor its recovery.

A 5 × 0.75 m experimental basin was constructed at the St. Anthony Falls Laboratory. Sediment and water were supplied continuously at a precisely controlled rate. The sediment and water were combined together in a mixing funnel before injection into the basin. The grain size of the supplied sediment was 0.12 ± 0.03 mm. The water discharge was 20 g/s, and the sediment supply was 0.6 g/s. A video camera recorded the evolution of the system. To visualize the river and monitor its depth, dye was supplied continuously during each videotaping session. After each videotaping session the dye supply was cut, and water flushed the dye from the system in a matter of a few hours. The recorded data were subsequently digitized for treatment and analysis. The studied region size was 0.75 × 1.0 m and was located between the 2.8 and 3.8 m marks (measured from the point of inflow). The final resolution of images was 3 mm across the river and 1.5 mm along the river.

The evolution of the braided river was characterized by monitoring changes in the pattern of the river and analyzing them according to the procedure described in section 2. It should be noticed that extracting the river patterns from the videotaped images presented significant difficulties, because very soon the sediment was colored with the same dye as water. However, extracting changes in the river patterns by subtracting images taken at different instants of time was found to be quite plausible, since the colored sediment patterns were almost the same and got zeroed when subtracted. Although changes in a river are three-dimensional, as is the river itself, we only considered in our previous work and here their horizontal projections. Thus changes (reflected by differences in the darkness of the images) represent water depth changes which include the cases of covering with water a previously dry area or exposing a previously water-covered area.

Before the experiment started, a certain initial amount of sediment was put manually in the basin such that a linear profile with a constant slope of 0.032 was formed (see Figure 2). This linear shape of the profile was different from the concave-shaped profile of the same average slope which would have been obtained had it been left to build up itself. Therefore the initially imposed shape of the profile can be considered as a vertical disturbance of the river. The profile of the river was monitored as it progressed. The initial profile and the profiles at the days when the video records of the river were taken are shown in Figure 2.

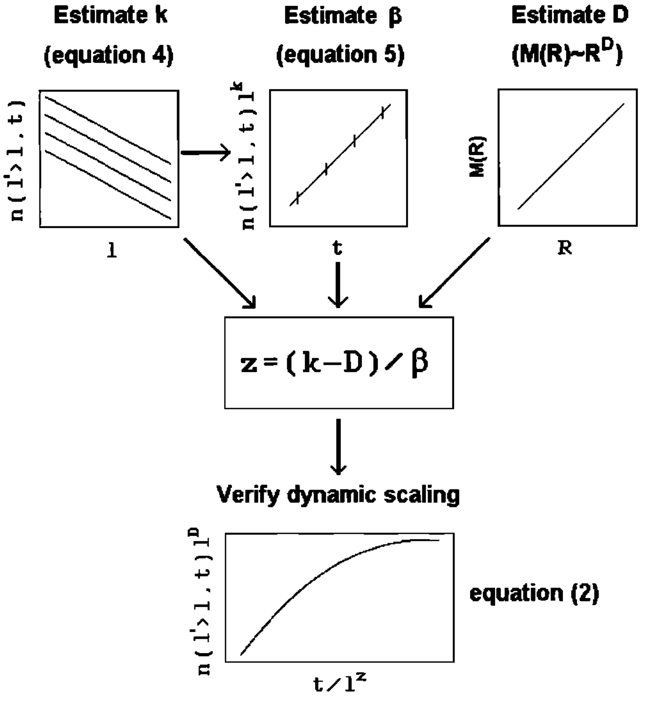

Figure 1. Schematic of estimation and verification procedure for dynamic scaling.

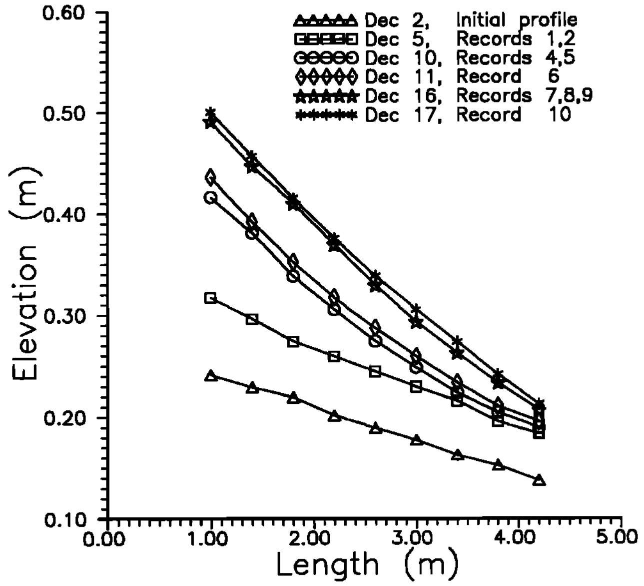

Figure 2. Evolution of the experimental braided river profile.

At several instances of time (with different profiles of the river) we manually wiped out the braiding pattern, thus introducing a horizontal disturbance. In these cases it took several hours for the river to recover the pattern. The process of recovering the braided pattern (horizontal self-adjustment of the river) was followed by recording the behavior of the river immediately after wiping out the braided pattern, after the pattern partially recovered, and after it recovered completely (the judgment on a complete recovery was made visually, by seeing that the degree of braiding was restored and that statistically, the braiding pattern did not change anymore). We qualitatively characterized the degree of braiding by attributing each pattern to one of three groups: "not braided" (immediately after the braiding pattern was erased), "half braided" (when the braiding was restored partially, usually about half an hour after the pattern was wiped out), and "fully braided" (several hours or more later). We did not characterize the degree of braiding more quantitatively, such as by computing the braiding index, because, as was mentioned earlier, the colored sediment did not allow accurate extraction of the braided river patterns, although it permitted accurate extraction of changes.

The process of recovery of the river from these disturbances was followed by recording the behavior of the river after the disturbances were introduced. We collected and analyzed the statistics of changes in the river at different days, as the river profile was building up and at different degrees of braiding. Table 1 displays the time the records were taken, the qualitative degree of braiding of the river for each record, and the average slope of the recorded region of the river (recall that this region had a length of 1 m and was located between the 2.8 and 3.8 m marks measured from the inflow point).

4. Results

First, at different profiles and different degrees of recovery of the braided patterns (records 1–10 in Table 1), the param-

Table 1. Video Recordings of the Experimental Braided River

Record ID	Day of the Experiment	Slope	Braiding
Record 1	3	0.037	none
Record 2	3	0.037	half
Record 3	4	0.041	full
Record 4	9	0.058	none
Record 5	9	0.058	half
Record 6	10	0.063	full
Record 7	15	0.080	full
Record 8	15	0.080	none
Record 9	15	0.080	half
Record 10	16	0.081	full

eter k was estimated from (4) using small values of the time lag t (4–32 s) and large enough values of l. The estimation was done only where appropriate, that is, where the $n(l' > l, t)$ versus l dependencies were a series of reasonably parallel straight lines in the log-log scale (Figure 3). Then from (5) the values of β were estimated (Figure 4). From the estimated values k and β the dynamic scaling exponent z was calculated using (6). Table 2 shows the estimated values of the parameters k, β, and z for different profiles and degrees of braiding. Absence of values for some records indicates lack of log-log linearity or parallelism in the $n(l' > l, t)$ versus l dependence even for small t/l^z values and therefore lack of dynamic scaling.

As mentioned above, (4) and (5) express scaling only for small values of the t/l^z parameter. They are convenient to use for estimating the dynamic scaling exponent z under the assumption that dynamic scaling is present in the system but by themselves are not enough to answer the question of whether dynamic scaling is indeed present. Only the general equation (2), which must be satisfied for all values of t/l^z, can answer this question. Therefore, for all records which satisfied (4) and (5) we plotted (using the estimated values of z) the values of $n(l' > l, t)l^D$ versus t/l^z, for different time lags from 4 to 256 s to test for each of them if the dynamics of the river satisfy the general equation of dynamic scaling (2) at any time lag. The value of D was estimated from a tracing of the river (using a mass-in-a-box method) as 1.7. Presence of dynamic scaling would be indicated by the fact that all $n(l' > l, t)l^D$ versus t/l^z curves for all time lags collapse to a single curve. As one can see in Plate 1, where the plots are presented, only records 7 and 10 reveal a reasonably good dynamic scaling. Note from Table 2 that these records correspond to the cases when the river was given enough time to develop the profile and to recover its braided pattern after it was wiped out. In all other cases when the river was not given enough time to either self-adjust its slope or to recover its braiding pattern after wiping it out, the dynamic scaling was not present. Thus the braided river paved its way to dynamic scaling through both vertical and horizontal self-organization.

5. Discussion and Conclusions

The main goal of this study was to provide more concrete evidence which would support or disprove our earlier hypothesis that braided rivers are self-organized critical systems [*Sapozhnikov and Foufoula-Georgiou*, 1997]. This hypothesis was based on our finding that in the statistical equilibrium state our experimental braided river showed dynamic scaling, and also on the fact that braided rivers are nonlinear systems with a high number of degrees of freedom, which is typical of SOC systems. However, there is one more crucial feature of critical systems: The quality of scaling in such systems depends on how close they are to the critical state [e.g., see *Ma*, 1976]. Therefore, to verify our hypothesis, we had to test whether the dynamic scaling really arises and improves with time, as the river undergoes self-organization. We believe that the results of this study showing that indeed dynamic scaling is not always present but develops only if the river is given enough time to self-organize strongly support our hypothesis and, together with the previous arguments, provide enough evidence that braided rivers are self-organized critical systems. Therefore they can be studied under the general framework of critical phenomena. This framework provides a conceptual basis for understanding critical systems as well as an apparatus for their description.

A distinct feature which makes braided rivers (together with some other SOC systems, such as a sandpile) essentially different from the traditional critical systems (such as magnetic materials) is the clear anisotropy between horizontal and vertical directions, produced by gravity. In fact, it was found in this study that the process of approaching the critical state by a braided river involves self-adjustment of both profile (vertical self-organization) and braiding pattern (horizontal self-organization). The theory of critical state of anisotropic systems is not developed yet. However, one can hypothesize that in contrast to the well-studied traditional isotropic critical systems, it takes tuning (self-tuning in the case of SOC systems) of more than one parameter to bring such systems to the critical state. Specifically, on the basis of the results of the experiment testifying that braided rivers exhibit dynamic scaling only after they undergo both vertical and horizontal self-adjustment, we conjecture that there are at least two self-tuning parameters which drive braided rivers to a critical state: One of them reflects vertical self-organization (profile self-adjustment), and the other accounts for horizontal self-organization (development of a braided pattern).

It should be noted that the two objectives, (1) revealing how the river self-organizes if it is left to evolve freely and (2) following how different factors affect the presence of dynamic

Table 2. Estimation of the k, β, and z Parameters of the Experimental Braided River

Day of the Experiment	Braiding: None	Braiding: Half	Braiding: Full
3, 4	record 1: ···	record 2: ···	record 3: $k = 2.63$, $\beta = 2.74$, $z = 0.34$
9, 10	record 4: ···	record 5: $k = 2.43$, $\beta = 2.63$, $z = 0.28$	record 6: $k = 2.12$, $\beta = 2.26$, $z = 0.19$
15, 16	record 8: $k = 2.00$, $\beta = 2.03$, $z = 0.15$	record 9: $k = 2.41$, $\beta = 2.32$, $z = 0.31$	record 7: $k = 2.49$, $\beta = 2.40$, $z = 0.33$ record 10: $k = 2.67$, $\beta = 2.34$, $z = 0.41$

Absence of values for some records indicates lack of log-log linearity and thus lack of dynamic scaling.

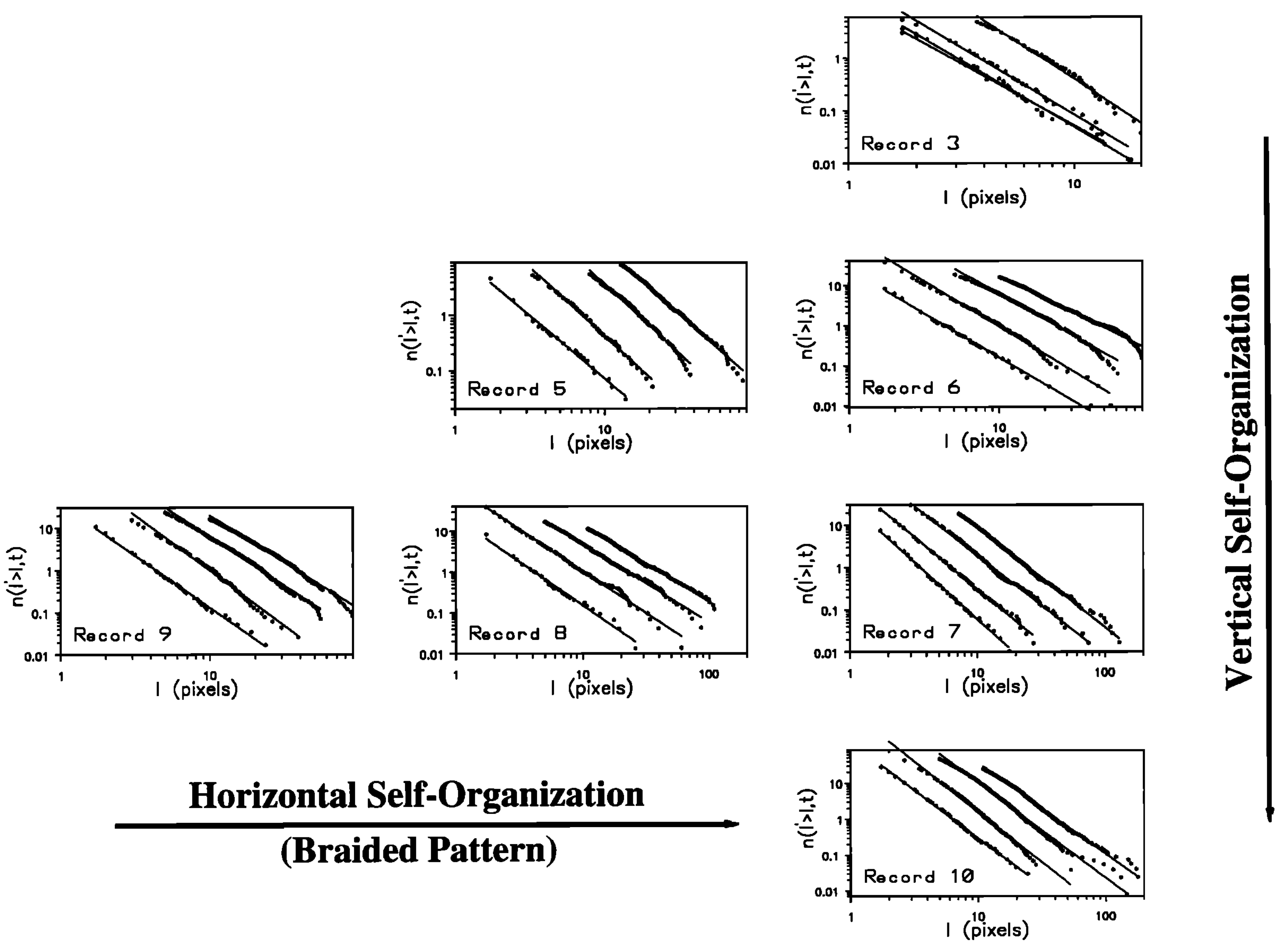

Figure 3. Estimation of the *k* exponent in the river (see equation (4)). For all records, except record 3, the curves are plotted for time lags *t* of 4, 8, 16, and 32 s (from bottom to top). For record 3 the time lags of 16, 32, 64, and 128 s are shown because the evolution of the river was slower owing to a lower slope.

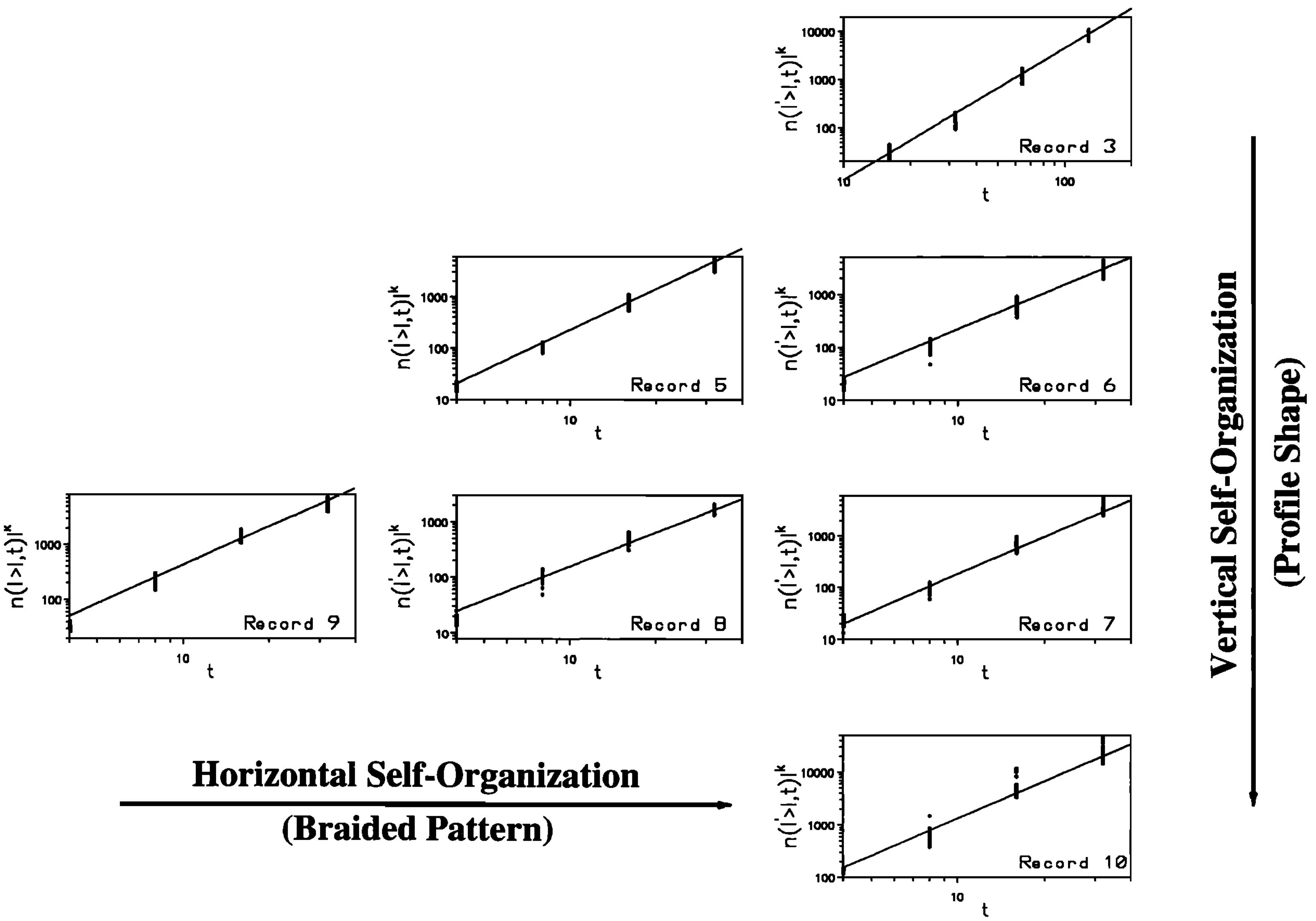

Figure 4. Estimation of the β exponent in the river (see equation (5)). Every vertically grouped set of points corresponds to one of the four distributions shown in Figure 3.

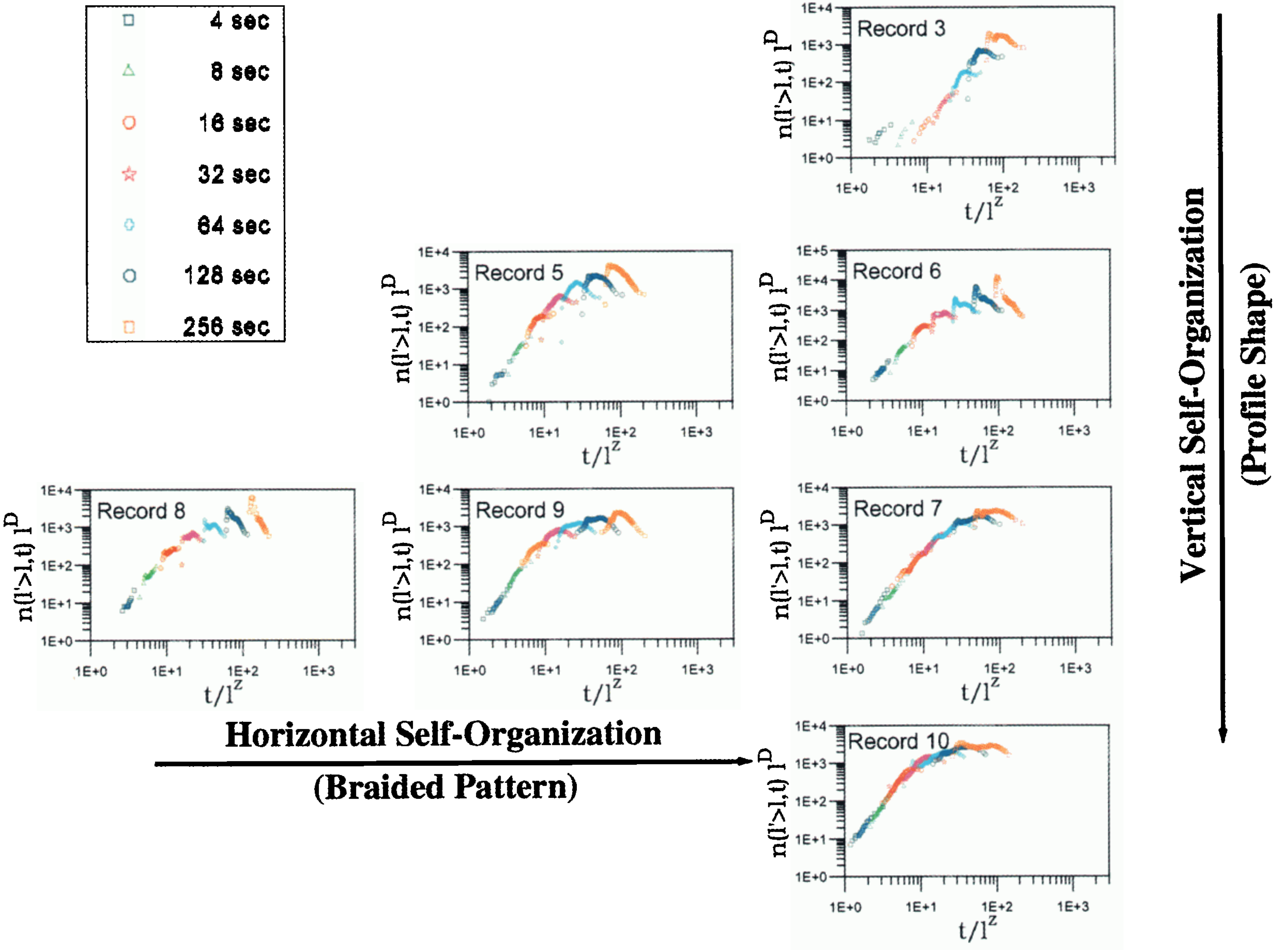

Plate 1. Verification of the presence of dynamic scaling in the experimental braided river according to the general equation (2). Observe how the dynamic scaling improves as the system undergoes vertical and horizontal self-organization.

scaling in the river, in a way conflict with each other from the point of view of their experimental implementation. This is because while the first one supposes that the river is left to evolve undisturbed, the second one requires introducing disturbances in the braided river system. This problem is common to all SOC systems which organize in a critical state themselves, and it can leave one with the dilemma to either (1) only observe the system approaching the critical state and not control the driving parameters or (2) control the parameters and accept the fact that the self-organization process is disturbed. However, one may be able to resolve this problem by separating the two processes in time, namely, by introducing short-time disturbances and then allowing for undisturbed self-organization in the system. In our case this was easier to implement for the horizontal than for the vertical self-adjustment. The reason for that is that the braided river was fairly shallow (deepest channels were just several millimeters in depth), and thus horizontal disturbances, that is, disturbances of the braided pattern, involved less bed material transport, even if the pattern was completely wiped out, and therefore required less time to recover than drastic vertical disturbances (significant changes of the profile). Thus we were able to introduce horizontal disturbances several times by erasing the braiding pattern and observe the river self-recovery afterward, while the vertical disturbance was imposed only once, by creating an initial profile which was far from the one which the river would have produced itself had it been left undisturbed.

It should be mentioned that on its way to the final equilibrium, and if left to develop without interference, the profile of the river undergoes an infinite series of relatively stable profiles. Although the profile keeps changing as the bed is built up, the change occurs significantly slower than the change caused by an artificially imposed profile disturbance of an arbitrary shape. Thus we call every naturally built profile a "quasi-equilibrium profile," in contrast to the profiles with the imposed disturbance. As the river evolved, it was recovering from the initial vertical disturbance by approaching a profile of a quasi-equilibrium shape, and at the same time it was developing toward the final equilibrium profile. This final equilibrium profile obtained by *Sapozhnikov and Foufoula-Georgiou* [1997] under the same conditions (same water and sand supply) is known to have a uniform slope of 0.15. We notice here, however, that by the time the dynamic scaling in the system had been achieved, the river was far from its final equilibrium profile. Indeed, as one can see from Table 1, the slope of the observed region was 0.08 when the dynamic scaling was achieved, which is approximately half the final equilibrium slope of 0.15. Therefore we hypothesize that the river reached the dynamic scaling because it reached a profile of a quasi-equilibrium shape (which continued to develop via other quasi-equilibrium profiles to the final equilibrium) and not because it approached closely enough the final equilibrium profile. Further experiments involving disturbances of the shape of several quasi-equilibrium profiles, as they approach the final equilibrium profile, are needed to answer this question conclusively. The disturbances would need to be small enough to enable introducing several of them before the final equilibrium profile is reached.

It has been long argued in the literature that relating scaling in natural objects to their physical properties is a high-priority task. The finding of this work that dynamic scaling in braided rivers arises only as they approach equilibrium (statistical equilibrium for the braided pattern and quasi-equilibrium for the profile) provides a highly desired connection between the scaling properties of the rivers and their physical state (closeness to equilibrium). Besides, it provides a quantitative tool of judging whether a braided river is close to equilibrium by checking its deviation from dynamic scaling. In a relatively short period of time one can collect information on the evolution of a natural river which would permit testing for the presence of dynamic scaling. The absence of dynamic scaling would imply that the river is far from equilibrium.

Although we presented here evidence that dynamic scaling arises in braided rivers only as they approach a stationary (critical) state, it is important for prediction purposes to be able to characterize quantitatively their deviation from the dynamic scaling behavior as a function of their deviation from stationarity. For that one would have to introduce two parameters describing the vertical and the horizontal self-organization of a braided river which would play the role of "temperature" in reaching the critical state of a traditional system. At this point we cannot be more specific other than suggesting that the first self-tuning parameter should reflect the shape of the profile (curvature and possibly the average slope) and the second one should depend on the braiding index. Considering the importance of old inactive channels as preferred pathways for active channel switching, an index which includes both active and inactive channels might be needed to describe the horizontal self-organization tuning parameter. The introduction of such two parameters would allow description of braided rivers which did not quite reach the critical state, a case which is often expected in natural systems. Indeed, it is known from the theory of the critical state [e.g., see *Ma*, 1976] that systems which are close to a critical state but not quite in it show scaling up to a certain scale (the correlation scale) which depends on the closeness of the system to the critical state. Thus description of such rivers in the dynamic scaling framework would still be possible, within, however, a more limited range of spatial and temporal scales.

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E. Foufoula-Georgiou and V. B. Sapozhkinov, St. Anthony Falls Laboratory, University of Minnesota, Mississippi River at Third Avenue, SE, Minneapolis, MN 55414-2196. (efi@mykonos.safhl.umn.edu; sapoz001@maroon.tc.umn.edu)

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