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On the accuracy of the maximum recorded depth in extreme rainstorms

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ABSTRACT The actual maximum depth of a storm is seldom measured but is usually approximated by the maximum recorded depth. These depths can be quite different depending on the raingage density and the spatial rainfall pattern. This paper presents a simplified analysis of the accuracy of the maximum recorded depth in extreme rainstorms.

INTRODUCTION

The peak rainfall depth occurring within a storm is a useful factor in the statistical analysis and classification of storms for hydrologic applications. It has been traditionally used in the area reduction formulae that convert storm center point rainfall to mean areal rainfall, usually assuming radial symmetry around the storm center (e.g., Woolhiser & Schwalen, 1959; Eagleson, 1967; Fogel & Duckstein, 1969; Boyer, 1957). The true peak of the storm, however, may be underestimated, depending on the gage density and the actual rainfall isohyetal pattern.

In a recent study, Richards et al. (1988) presented an investigation of an extreme but unusually well documented storm near Smethport, Pennsylvania in July, 1942. They showed that analysis of the storm based on the standard observation network underestimated the actual rainfall depths (measured from bucket-surveys) by as much as 200%. For example, the maximum depth measured by the standard network was found to be 18 inches while bucket surveys indicated that the actual maximum depth was close to 34 inches. Although this might be an unusual storm, other studies have found that underestimation, of up to 60%, of maximum depths is not unusual. For example, Huff et al. (1958) have reported that the ratio of field survey to climatological network maximum point depths for extreme storms in Illinois during the period of 1948-1957 had an average value of 1.28 and a maximum value of 1.62. As the averaging area around the storm center increases, the error of mean rainfall depth decreases as is shown in Figure 1, plotted from data given in Huff et al. (1958). In that figure, $\bar{d}_a(A)$ denotes the "actual" average depth over an area A centered around the storm center, and $\bar{d}_m(A)$ denotes the "measured" (or actually, the estimated from measured data) average depth over the same area. $\bar{d}_a(A)$ was estimated from a very dense network of field survey data and $\bar{d}_m(A)$ from the standard National Weather Service (NWS) rainfall network.

Ideally, one would like to know the distribution, or at least the first few statistical moments, of the error of the maximum value of the rainfall field as a function of the raingage density and the spatial distribution of rainfall. (Error is defined as the difference between the actual maximum and the measured maximum depths of the storm.) This is a difficult problem, however, for which analytical solutions cannot easily be obtained (e.g., Vanmarcke, 1983, Ch. 4). Here, we approach the problem in a simplified framework, in the sense that we consider the rainfall field as being described by a deterministic spread function of a known functional form and parameters, but with center occurring randomly within the raingage network. The results of this oversimplified theoretical error analysis are in good agreement with the experimental results of Huff et al. (1958). This suggests that for all practical purposes, such an approach may be adequate since, after all, the spatial distribution of the rainfall field at the vicinity of the maximum seems to be well approximated with a spread function of circular or elliptical, geometrically similar contours.

This study was motivated by the need to quantify the inconsistencies among storms measured from raingage networks of varying density, and develop methods of adjustments. Such adjustments may be appropriate in storm regionalization and storm transposition studies (e.g., Foufoula-Georgiou, 1988) where it is known that the raingage networks have changed considerably over the period of record and also differ in density from region to region.

STATISTICAL MOMENTS OF THE ERROR IN THE MAXIMUM RECORDED RAINFALL

Let d_o denote the *actual maximum* depth at the storm center and d_m the *maximum measured* rainfall depth at a station close to the storm center. Our interest is in deriving the moments of the normalized error

$$e = (d_o - d_m)/d_o \tag{1}$$

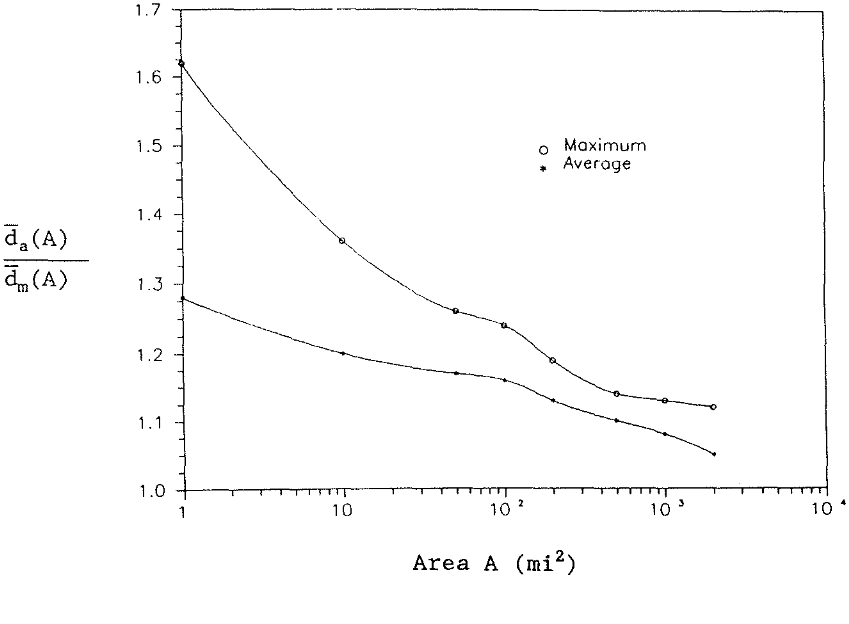

Fig. 1. Ratio of actual average depth ($\bar{d}_a(A)$) versus measured average depth ($\bar{d}_m(A)$) as a function of storm area A (mi^2). [Plotted from data given in Huff et al., 1958].

Assuming that the storm isohyetal pattern is circular and that the raingages are evenly distributed at the vicinity of the storm center (Fig. 2), d_m is the depth at the station closest to the actual storm center. Let (2h) denote the distance between any two raingages, and r_{cg} the distance between the actual storm center and the raingage closest to this center. Under the hypothesis of a uniform spatial distribution of the storm center within the triangle formed by the three nearby raingage stations, the m-th moment of r_{cg} is given by

$$E(r_{cg}^m) = \frac{1}{A_t} \int_{x=0}^{x=h} \int_{y=0}^{y=x/\sqrt{3}} (x^2 + y^2)^{m/2} \, dy \, dx \tag{2}$$

where A_t is equal to one sixth of the area of the triangle formed by the raingages (Fig. 2).

Evaluation of the above integral results in the following expressions for the first four moments

$$E(r_{cg}) = (\sqrt{13}/3\sqrt{3})h = c_1 h \tag{3a}$$

$$E(r^2{}_{cg}) = (5/9)h^2 = c_2 h^2 \tag{3b}$$

$$E(r^3{}_{cg}) = \frac{\sqrt{3}}{10}\left(\frac{17}{9} + \frac{3}{2}\log\sqrt{3}\right)h^3 = c_3 h^3 \tag{3c}$$

$$E(r^4{}_{cg}) = (56/135)h^4 = c_4 h^4 \tag{3d}$$

Higher moments can be computed but the procedure is tedious. Of course, the use of the first four moments only, limits the range of applicability of the analytical results as will be discussed in the sequel.

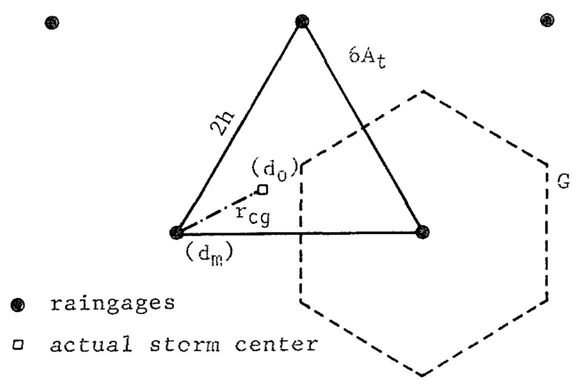

Fig. 2. Illustration of the raingage network and the area covered by each raingage.

Having derived the moments of the distance between the actual peak and the maximum recorded depth, one has to specify the spread function describing the storm in order to derive the moments of the normalized error e. Although it is recognized that, in general, the spatial distribution of rainfall depths within the storm is fairly complex, the distribution of rainfall depths within the core (or eye) of the storm is more well behaved and approaches an unimodal decay spread function with approximately circular or elliptical geometrically similar contours around the maximum storm depth. Based on this argument, we concentrate here, as a first approximation, on a deterministic simple spread function of the form (Horton, 1924)

$$\bar{d}(A) = d_o \exp(-kA^n) \tag{4}$$

where d_o is the actual depth (inches) at the storm center, $\bar{d}(A)$ is the average depth over a storm area A (mi^2), and k and n are fitted parameters. From this equation, one can derive the relationship for the depth along an isohyet enclosing an area A as

$$d(A) = d_o\, e^{-kA^n} (1 - knA^n) \tag{5}$$

Assuming a circular storm, the depth along an isohyet at distance r from the center is

$$d(r) = d_o\, [1 - kn\pi^n r^{2n}] \exp(-k\pi^n r^{2n}) \tag{6}$$

Horton (1924) gives a nice meteorological explanation supporting the multiplicative form of the above empirical relationship. Court (1961), Boyer (1957) and Horton (1924) give ranges of the parameter values k and n for several types of storms reported in the literature. For example, Horton (1924) has found that for small duration intense storms in Boston over 20 mi^2 n varied between 0.53 and 0.61. For one-day storms in the eastern half of U.S. over areas of 20,000 mi^2, n varied between 0.33 and 0.56 with an average value of 0.45. A recent study of 77 extreme midwestern storms (Foufoula-Georgiou & Wilson, 1988) found that the parameter n had a mean value of 0.48 and a standard deviation of 0.12. (These estimates were obtained from a weighted least squares fit of (4) to the maximum 24-hour depth-area data for storm areas enclosed within contours of depth greater than or equal to 3 inches.)

To simplify the computations while illustrating the order of magnitude of the relative error e for some realistic spread functions, we assume here a fixed value of n close to its expected value, n = 0.5. This assumption simplifies (6) to

$$d(r) = d_o\, (1 - 0.5\lambda r)e^{-\lambda r} \tag{7}$$

where $\lambda = k\pi^{0.5}$.

Note that the depth of interest d_m is the depth at distance r_{cg}, i.e., $d_m = d(r_{cg})$. By expanding (7) in Taylor series and ignoring the effects of fifth or higher order terms we obtain

$$E(d_m) \approx d_o\, [1 - 1.5\lambda E(r_{cg}) + \lambda^2 E(r^2_{cg}) - (5/12)\lambda^3 E(r^3_{cg}) + (1/8)\, \lambda^4 E(r^4_{cg})] \tag{8}$$

and

$$E(d^2_m) \approx d^2_o \, [1 - 3\lambda \, E(r_{cg}) + (17/4) \, \lambda^2 \, E(r^2_{cg}) - (23/6) \, \lambda^3 \, E(r^3_{cg}) + (15/6) \, \lambda^4 \, E(r^4_{cg})] \quad (9)$$

If we let G denote the raingage density (i.e., 1 station per G mi^2), then

$$G = 2\sqrt{3} \, h^2 \quad (10)$$

Introducing (3a-d) into (8) and (9) and using (1) and (10) one obtains

$$E(e) \approx m_1 \, k \, \sqrt{G} - m_2 k^2 G + m_3 k^3 G^{3/2} - m_4 k^4 G^2 \quad (11)$$

where

$$m_1 = 1.5 \, \sqrt{\beta} \, c_1 \quad (12a)$$

$$m_2 = \beta \, c_2 \quad (12b)$$

$$m_3 = (5/12) \, \beta^{3/2} \, c_3 \quad (12c)$$

$$m_4 = (1/8) \, \beta^2 \, c_4 \quad (12d)$$

c_1, c_2, c_3, c_4 have been defined in (3) and $\beta = \pi/(2\sqrt{3})$. Similarly, the variance of e is computed as

$$\begin{aligned} var(e) \approx{} & (m_5 - m_1^2) \, k^2 G - (m_6 - 2m_1 m_2) \, k^3 G^{3/2} \\ & + (m_7 - m_2^2 - 2m_1 m_3) \, k^4 G^2 + (2m_1 m_4 + 2m_2 m_3) k^5 G^{5/2} \\ & - (m_3^2 + 2m_2 m_4) \, k^6 G^3 + 2m_3 m_4 \, k^7 G^{7/2} - m_4 k^8 G^4 \end{aligned} \quad (13)$$

where m_1, m_2, m_3, m_4 have been defined previously and

$$m_5 = (9/4) \, \beta \, c_2 \quad (14a)$$

$$m_6 = 3 \, \beta^{3/2} \, c_3 \quad (14b)$$

$$m_7 = (27/12) \, \beta^2 \, c_4 \quad (14c)$$

Observe that equations (11) and (13) are both of the form

$$\sum_{i=1}^{\infty} c_i k^i G^{i/2} \quad (15)$$

Because the derivation has considered only up to fourth moments not all coefficients c_i in (13) are complete. Also, the range of applicability of these equations is limited by the rate of decay of the term $(kG^{1/2})^i$ as i increases. In other words, $kG^{1/2}$ must be much less than one so that the omitted higher order terms in the summation of (15) are negligible. Fortunately, the value of k for most extreme storms is of the order of 0.01 to 0.05 and G is of the order of 20 to 50 mi^2 since supplemental data (i.e., bucket surveys) are usually available for extreme storms.

In the above derivation, it was assumed that the raingage network is fixed and has the triangular configuration of Fig. 2. Actual networks are more irregular and the above results would only hold approximately. However, they should provide fairly close approximations, if one observes, for example, that whereas the mean distance between a randomly chosen point within a triangle of unit area and the vertex of the triangle closest to that point is 0.5272 (as computed from (3a) for $h^2=1/\sqrt{3}$), the mean distance between two randomly chosen points in an equilateral triangle of unit area is 0.5544 (Matern, 1986).

APPROXIMATION OF THE ERROR OF THE MAXIMUM DEPTH IN EXTREME RAINSTORMS

It is important to note that the above analysis considered the storm as being a single-center storm with radial symmetry. This is not the case, however, for all extreme storms as often more than one "cells" or centers exist. The effect of the single center (which is implicit in the use of the Depth-Area-Duration curves for the estimation of the parameters n and k), is that the fitted spread function exhibits a much more slowly decaying pattern as compared to the actual spread function of the individual cells. It is interesting to observe, however, that studies of the spatial pattern of raincells (e.g., Konrad, 1978; Bonser, 1986) have found that similar relationships also apply at this smaller scale of spatial rainfall distribution. For example, Bonser (1986) analyzed a total of 705 raincells from 24 radar maps representing twelve hours of storm data within 120 km of the radar site. The resolution of the data was a 2x2 km grid. Drawing a comparison between our spread function and that used by Bonser, the average n value of the raincells was 0.513, suggesting as before that a relationship of the form of $d(r) \propto e^{-kr}$ seems adequate. Of course, differences between the k values of the total storm and the k value of the cells are expected. To get an idea of how these k values compare with each other, the following simple, illustrative example is considered. Suppose that a storm is composed of two identical cells of center depth equal to d_o and parameter values (k', n') in the relationship of equation (4). The total average depth-area relationship for this storm will consider these two cells lumped together to one cell with the same center depth d_o and with parameter values (k,n). If we assume that n'=n, as empirical evidence suggests, it is easy to show that $k=k'/2^n$. For the value of n=0.5 one obtains k'=1.41 k, and for n=0.3 k'=1.23k.

The implication of the above analysis is that the errors in the maximum recorded rainfall derived using the published depth-area data will in general be more conservative than the actual errors, simply because the lumping of the raincells together slows down the decay of the total spread function. Thus, for example, the average value of k=0.04 found for the extreme midwest storms (Foufoula-Georgiou & Wilson, 1988) may correspond to a larger k value for the individual raincells.

To get an idea of the order of magnitude of the mean error, note, for example, that if the raingage network had density of 1 station per 50 mi^2, the maximum actual depth of a storm with spread function with n=0.5 and k=0.01 would be underestimated by approximately 7%, while the peak of a storm having n=0.5 and k=0.05 would be underestimated by an average of 30% (Fig. 3). For a frame of comparison with experimental results, it is interesting to note that Huff et al.(1958) found an average underestimation of the peak storm depth of up to 19% when the maximum recorded values by the standard NWS networks were compared with those of a more dense experimental network.

Due to the limit of applicability of the analytical results to n=0.5 and small values of k and G, the first three moments of the normalized error have been computed via simulation for other values of n and larger values of k (Fig. 4). Figure 4 also compares the first three moments of e for a circular and an elliptical storm of major to minor axis ratio equal to 2, when both storms have the same average depth-area relationship. The elliptical shape with ratio 2:1 was found to be the most frequent shape of midwestern storms by Huff (1968) and also Foufoula-Georgiou & Wilson (1988). It is observed that the error is slightly larger and much more variable when the raincell has an elliptical shape as compared to one with a circular shape.

CONCLUSIONS

The difference between the actual maximum depth of a storm (d_o) and the maximum recorded depth (d_m) may be appreciable depending on the storm's spatial pattern and the raingage density. In this paper, we have presented a simplified error analysis of d_o and have estimated the first three moments of the normalized error $e = (d_o - d_m)/d_o$ as a function of the raingage density. Such results may be useful in devising storm depth adjustment procedures when storms of different spatial patterns and variable density recording networks are combined in storm regionalization or storm transposition studies.

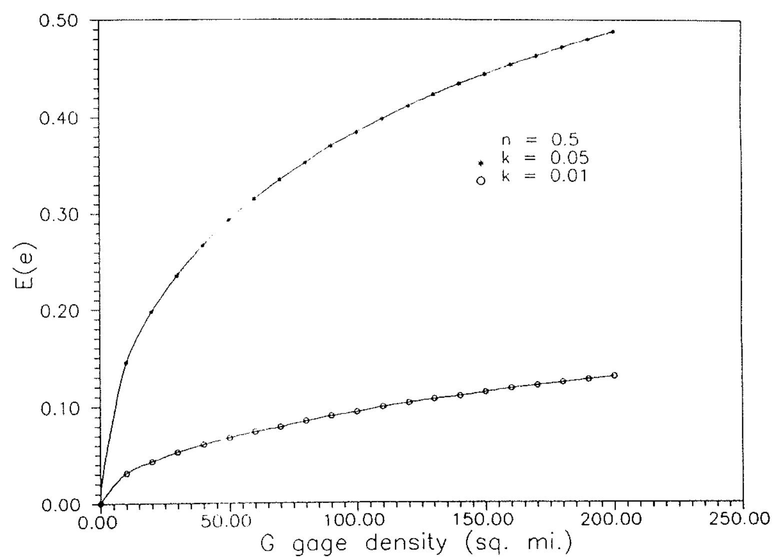

Fig. 3. Mean normalized error (derived analytically) as a function of the raingage density.

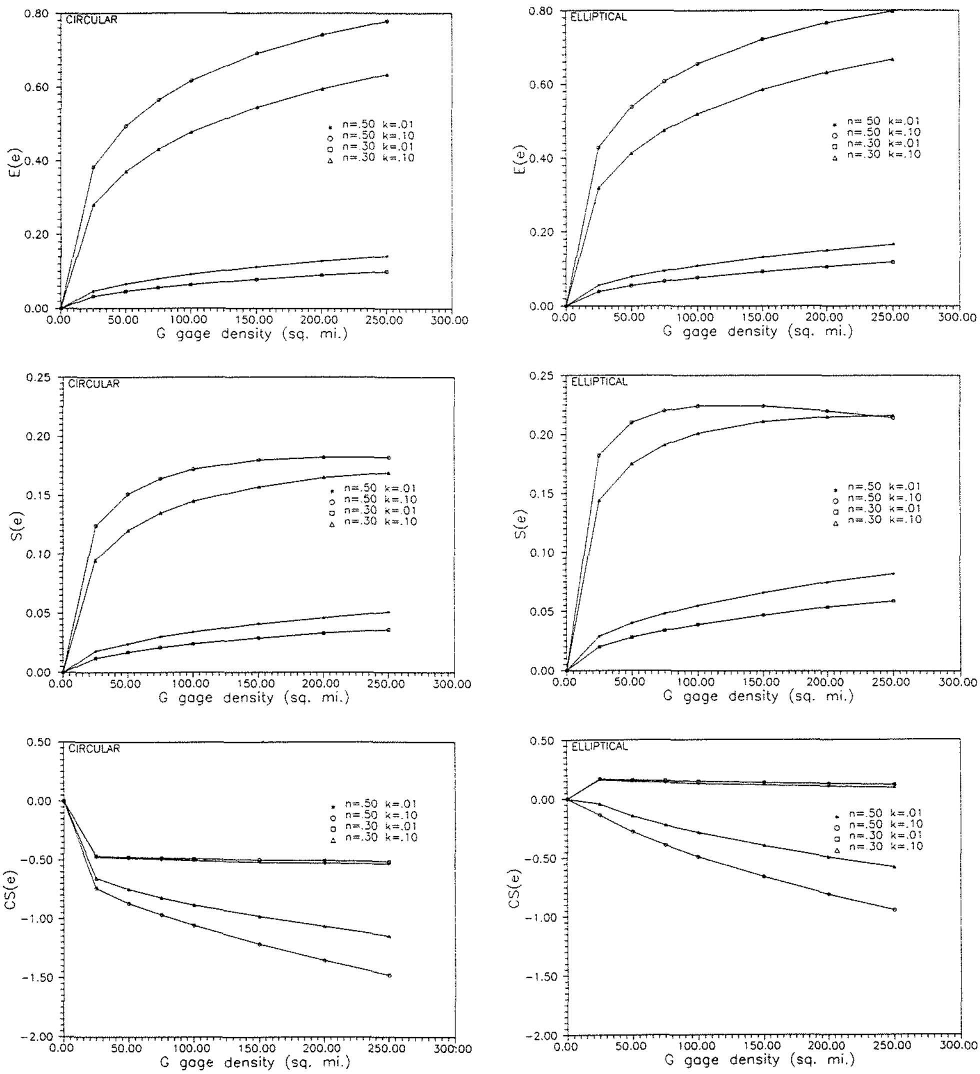

Fig. 4. Mean, standard deviation, and skewness coefficient of the normalized error as function of the raingage density. Comparison of circular and elliptical storms with the same depth-area relationship. Results derived via simulation.

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